STATISTICAL MODELLING OF THE INDOOR RADIO CHANNEL – AN ACOUSTIC ANALOGY

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INTRODUCTION

Many models exist for the prediction of the wideband characteristics of the indoor radio channel. Techniques range from simple empirical models, to complex site-specific algorithms making use of ray-tracing or finite difference methods.

The simpler models offer little guidance to the designer of practical systems, while the more complex approaches generally require a level of knowledge of the physical characteristics of the building that is at best time consuming and at worst, impossible, to obtain.

This paper proposes an intermediate approach to the modelling of the indoor, wideband, radio channel, following a physical-statistical approach initially inspired by methods employed in acoustic modelling.

THE MODELLING OF ROOM ACOUSTICS

The majority of the modelling of room acoustics is performed, nowadays, using ray-tracing models analogous to those employed by radio system modellers, and the information required in both fields is similar (amplitude plots and information on channel time-delay). The field of acoustic modelling is, however, clearly much older than its radio counterpart, and the computer power necessary to undertake such detailed modelling has only recently been widely available.

Prior to the development of such ray-tracing methods, the acoustic community typically made use of models which captured the bulk statistical behaviour of rooms (though full wave solutions were developed for relatively simple cases). Much of the theory of such models stems from the work of Sabine [1] in the early 20th Century.

Sabine equation for reverberation

The Sabine model for the decay of sound in a room can be derived by noting that the rate of change of acoustic energy, *w*, in a room is the difference between energy supplied and energy leaving the room.

For a diffuse field, geometrical considerations lead to simple expressions for these energies, leading to a differential equation, the solution of which gives the 'Sabine equation':

$$w(t) = e^{\frac{-t}{\tau}} \qquad (1)$$

Where τ (s) is the *'characteristic decay time'* of the room, defined as:

$$\tau = \frac{4V}{c\alpha S} \qquad (2)$$

in which V and S are the volume and surface area of the room (m), c the speed of sound (m/s) and α an absorption coefficient.

Modifications to the Sabine model

If a room is highly absorptive, the Sabine formulation breaks down: for a perfectly anechoic room, the model gives a non-zero decay time. A modification was proposed by Eyring [2].

This model considers explicitly the amount of energy lost at each reflection of a sound ray. The rate of reflection, and hence the rate of loss, is determined by the '*mean free path*' (*mfp*) of the room, the average path length between reflections, given by:

$$l_{mfp} = \frac{4V}{S} \qquad (3)$$

where V and S represent, respectively, the volume and surface area of the room (see discussion below).

The characteristic time in this formulation is given by:

$$\tau = \frac{l_{mfp}}{-v \ln(1-\alpha)} \qquad (4)$$

Application to radiowave propagation

Given the background outlined above, it seems attractive to apply some of the same techniques to the radio case. At least one proposal has been made that involves applying modelling techniques involving bulk room characteristics to radiowave propagation.

In [3], a model is developed for the average power delay profile characteristics of a room. This model makes two simplifying assumptions:

(i) The power delay profile of the channel is modelled as the usual set of delay 'bins', each of duration τ_c, the '*characteristic time*' of the room, defined as the time for a set of rays to suffer a single reflection. This is related to the mean free path of the room by:

$$\tau_c = 2 \cdot \frac{l_{mfp}}{c} = \frac{8V}{cS} \qquad (5)$$

It is assumed that <u>all</u> rays suffering *n* reflections (i.e. n^{th} order rays) will arrive in the bin centred at a delay

$$\tau_n = \frac{\tau_c}{2}(2n-1) \qquad (6)$$

(ii) The set of rays arriving in a given bin, having suffered n reflections, will be attenuated by a factor R^n, where R is the average power reflection coefficient for the room, calculated by averaging over all angles of incidence and all surfaces. The reciprocal of this reflection coefficient is equivalent to absorption corefficient, α, used in (2) and (4).

The decay rate predicted by this model demonstrated a fair fit to the envelope of the PDP measured in two rooms at 1.5 GHz.

This paper proposes an enhanced model that will take greater account of the spread of arrival times of reflected power; further consideration has also been given to the estimation of the mean free path of a room.

PROPOSED NEW MODEL

In [3] the assumption is made that all power associated with a given number of reflections arrives in a single time delay 'bin'. In practice, this will not be the case, and an improvement in the accuracy of the model might be expected if a more accurate account is taken of the distribution of arrival times.

Multipath arrival times are often characterised by a Poisson distribution, or a variant thereof such as the 'Δ-K' model [4]. For the initial model presented here, a simple Poisson distribution is assumed, with, for instance, the mean arrival rate of first-order rays of a given length corresponding to the ratio of ray length to mean free path.

Figure 1 illustrates the assumed likelihood of energy arriving at a receiver at a given path length (and hence delay time) after suffering between 1 to 4 reflections.

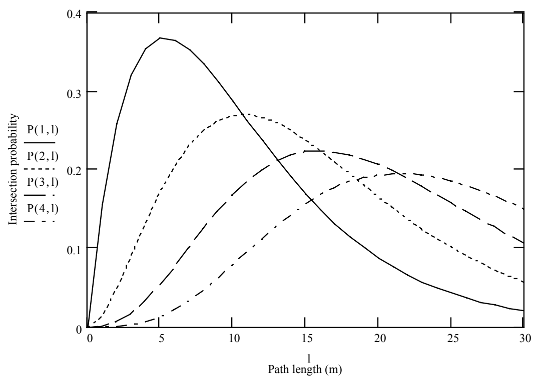

Figure 1: Probability of multipath arrival (by order)

In the proposed new model, the sum of multipath and direct power at the receiver, for a given excess delay, τ, is given by:

$$P_{rx}(\tau) = P_0 \cdot D_\tau^{-2} \sum_n (Q_{reflection}(n,\tau,l_{\text{mfp}}) \cdot R^n) \quad (7)$$

where P_0 is the power received at 1m, D_τ the path length (m) corresponding to a given excess delay, *n* represents reflection order, and $Q_{reflection}$ describes the pdf of arrival probability. The value of $Q_{reflection}$ for $n=0$ is a Dirac function at a delay time corresponding to the terminal separation.

The inputs required for the model are:

- Bulk room geometry (surface area, volume)
- Average room reflection coefficient (or complex permittivity of surfaces & frequency)
- Transmit-receive distance

The output of the model can be in the form of either a single value of overall received power, or a power delay profile, at a given location.

The initial version of the proposed new model has a number of limitations; only a single room is currently modelled, and both transmit and receive antennas are assumed to be isotropic. The structure of the model, however, should allow the additional modelling (in physical-statistical terms) of a number of other effects, such as room clutter.

To apply (7), it is necessary to determine, from the input parameters, values for the mean free path of the room and an average reflection coefficient. The derivation of these values is now considered.

Determination of Mean free path

The starting point of the model is to determine the 'mean free path' of the enclosure/room under consideration.

In acoustics, the mean free path (*mfp*) of an enclosure is often stated, without further qualification, as being given by (3) above. In some of the papers in which the concept has been applied to EM wave propagation this deceptively simple formulation has been applied without further examination.

A number of equivalent derivations for *mfp* have been given [5,6], but all make the assumption that energy scattered from the walls follows a Lambertian law (i.e.

$$P_s(\theta) = \frac{\rho}{\pi} P_i Cos(\theta) \quad (8)$$

where ρ is the reflection coefficient, P_i and P_s the incident and scattered powers respectively, and θ the scattering angle)

The expression above gives the average value of mean free path for a large number of rays of all possible directions of propagation. It will only apply to a single ray if the reflection mechanism is diffuse – i.e. the ray suffers random changes in direction due to random scattering. Only if this condition is met can we use *mfp* in determining the decay time of acoustic energy.

For the radio case it is (generally) not the case that the time average *mfp* is equivalent to the ensemble average *mfp*, as reflections are not diffuse. A series of Monte-Carlo simulations have been undertaken, to explore the way in which these quantities vary with room geometry.

Three hypothetical rooms are modelled in this exercise, with the characteristics shown in the table below.

	Ratio	**LxWxH (metres)**
Room A	1:10:10	20 x 20 x 2
Room B	1:10:1	2 x 20 x 2
Room C	1:1:1	4 x 4 x 4

Table 1: Room dimensions used in modelling

Two forms of ray-tracing simulation have been undertaken. In the first ('random'), rays are launched from a large number of arbitrary positions, in arbitrary directions. The progress of each ray is followed up to a given order of reflection, with the length of each segment recorded.

In the second type of simulation ('constrained'), random positions are generated for both receiver and transmitter, and a standard ray-tracing procedure between the two is followed (a ray-launching algorithm is used). Again, all transmit-receive paths up to a given order are determined, and the statistics logged as before.

The results of these simulations are presented in Figures 2 and 3 (at end of paper). In each case, the results of a previously published [5] set of Monte-Carlo simulations for the diffusely-scattered acoustic case are indicated for comparison. In these acoustic simulations, a ray was launched in a specific direction, and path lengths recorded over a very large number of intersections. At each intersection with a surface, the reflection angle was taken randomly from a distribution corresponding to the Lambertian law (8).

It can be seen that in the cases of rooms 'A' and 'B' (which have extreme aspect ratios) the results obtained using the constrained algorithm are very different from the 'random' case (and from the acoustic case). This is not so for the case of the cubical room 'C'. The difference is due to the fact that, in the 'constrained' case, the rays whose lengths have been recorded, will, statistically, be aligned more often with the longest axis of the rooms, reflecting the limitations on possible terminal positions.

The results of each algorithm were also used to derive values of mean free path for each room, and these are tabulated below, together with the calculated value from (3) above.

metres	***mfp***	**'random'**	**'constrained'**
Room A	3.333	4.074	7.268
Room B	1.905	2.026	4.417
Room C	2.667	2.548	2.219

Table 2: Comparison of *mfp* values

Again, it can be seen that, for the rooms with extreme geometries, the calculated mean free path is significantly different from that obtained by the 'constrained' simulation.

It is, therefore, arguably the latter that should be applied in any simulation; however, there seems to be little point in performing a computationally-intensive simulation simply to obtain this parameter. Having undertaken a ray-trace simulation, it would clearly be ridiculous to discard the detailed results to perform a simpler and less accurate prediction! It will be necessary to develop an analytical method for the estimation of mean free path in a scenario in which specular reflection is significant, and in which ray directions are not random, but constrained by terminal positions.

Other statistics have also been gathered from the MC simulations. A systematic increase in delay spread with transmitter-receiver separation is often reported. The 'constrained' simulation was configured to record the mean free path for each randomly-generated transmitter-receiver pair. The results are indicated below:

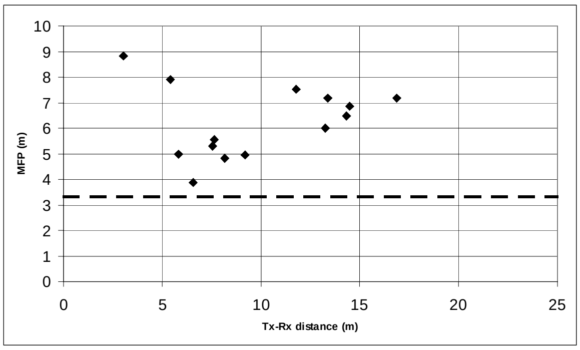

Figure 4: mfp vs. terminal separation, room A

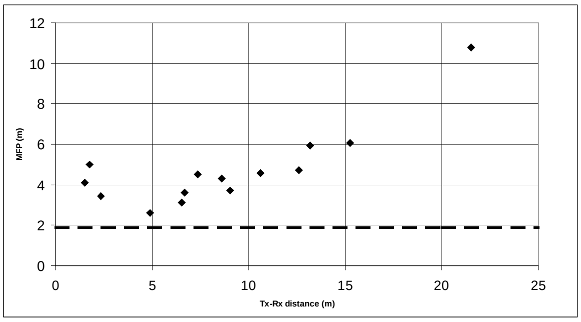

Figure 5: mfp vs. terminal separation, room B

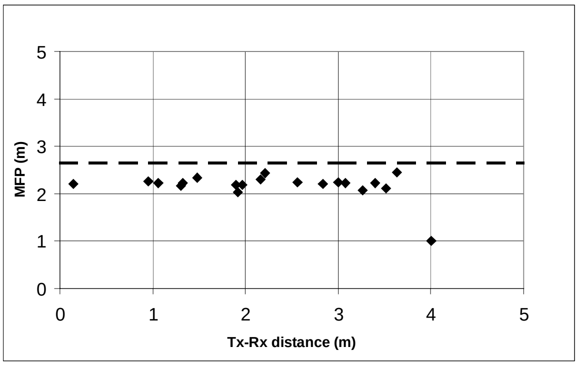

Figure 6: mfp vs. terminal separation, room C (m)

The tendency for the mean free path to increase with terminal separation is clear except in the case of the symmetrical room 'C'. For asymmetrical rooms, longer paths imply that the direction of transmission is increasingly aligned with the longest dimensions of the room.

Any new analytic expression for mfp should therefore take as a parameter, not only information on room geometry, but also the terminal separation.

Determination of reflection coefficient

In a diffuse-scattering environment, a single value for reflection coefficient may be obtained by averaging over all surfaces and scattering angles. This may no longer be the case where specular reflection is significant. The form of the proposed model, will allow the value of assumed coefficient to vary, not only with terminal separation, but also with reflection order. In particular, modification of the 'effective' reflection co-efficient with path length may allow antenna directivity to be accounted for.

CONCLUSIONS

A new model has been proposed for the prediction of average radio channel power delay profile characteristics of a room.

Where previous models have assumed, for each time delay bin, a binary arrival probability for ray arrivals of each order, the new model allows the continuous distribution of ray arrival statistics to be modelled.

Particular attention has been drawn to the interpretation of 'mean free path', a parameter often used in both acoustic and radio predictions of indoor propagation. The different statistics of mean free path pertaining to scenarios dominated by diffuse scattering and specular reflection have been explored using Monte Carlo simulation.

REFERENCES

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3. Holloway, C.L. "A model for predicting the power delay profile characteristics inside a room", IEEE Trans. Veh.. Techn, 48, 4, July 1999.

4. Turin, G.L., "A statistical model of urban multipath propagation", IEEE Trans. Veh. Techn., 21, pp1-9, Feb 1972

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6. Pierce, A.D., "Acoustics: an introduction to its physical principles and applications", pp.260-262, McGraw Hill, 1981

Figures 2 & 3: Simulation results

In all plots below, the x-axis represents path length, normalised to the mean free path calculated by equation (3).

The dashed curves indicate the values obtained in the acoustic simulations reported in reference [5].

Figure 2: 'Free' algorithm

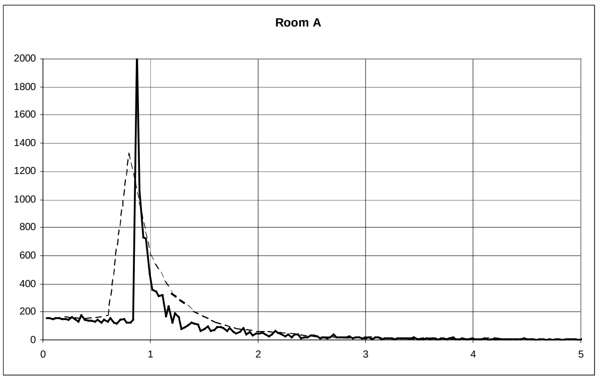

Figure 2a: Room A

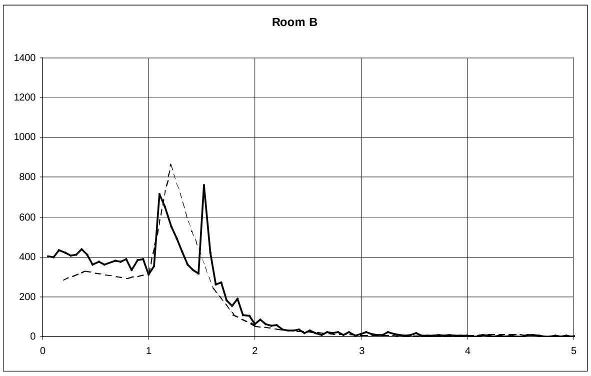

Figure 2: Room B

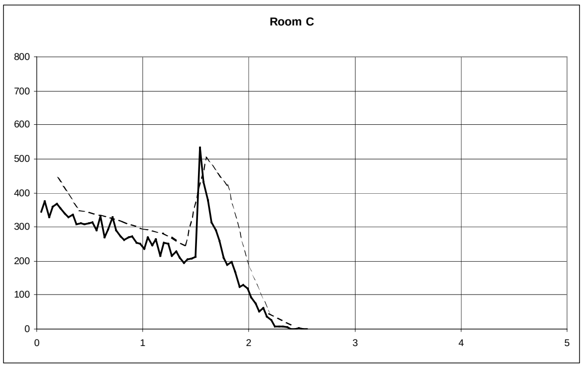

Figure 2: Room C

Figure 3: 'Constrained' Algorithm

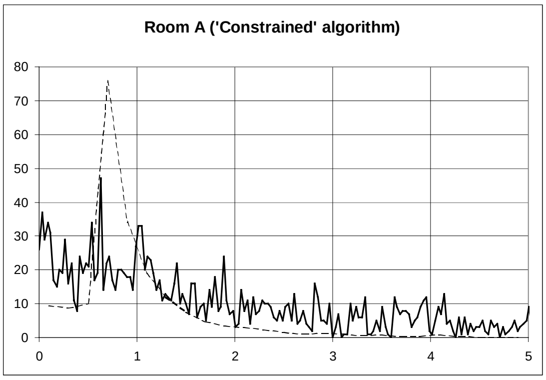

Figure 3: Room A

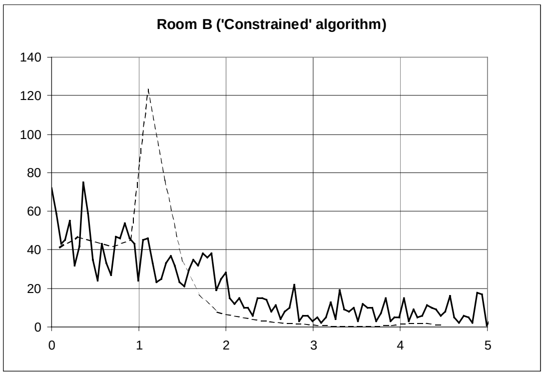

Figure 3: Room B

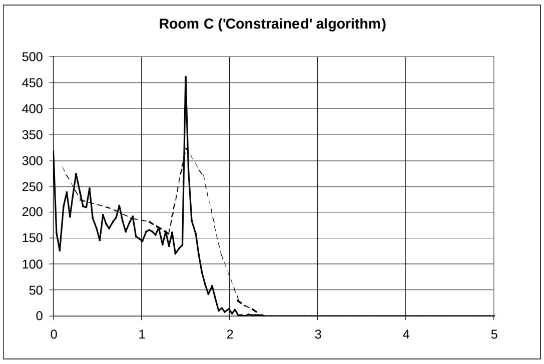

Figure 3: Room C