Rapid, uniform computation of multiple knife-edge diffraction

C. Tzaras and S.R. Saunders

A new formulation for the evaluation of the diffraction attenuation function for arbitrary numbers of knife-edges is presented, being useful for predicting radio propagation over buildings or hills. The outcome is characterised by rapid evaluation while obtaining accurate predictions for any path profile, including cases where other models fail.

Introduction: In propagation modelling for fixed or mobile radio systems, diffraction is the dominant propagation mechanism when a significant number of obstacles exists between a transmitter and a receiver, as shown by [4]. The representation of such obstacles by infinite knife-edges has been found to be appropriate for built-up areas [2], with the Vogler solution [1] representing the ultimate in accuracy compared with other approximate deterministic models [3, 4]. However, the Vogler solution has been found to be impractical when a large number of edges (say > 5) is included in the path since either it fails or the computational requirements are prohibitive. Although Vogler's solution can become more efficient by considering the significant knife-edges only (for example those which lie inside the first Fresnel zone), such an approach removes the sensitivity of the method in arbitrary path profiles and also leads to unreliability, since the performance of the method becomes unpredictable. An alternative method [5] has been found to overcome these difficulties, but only when two knife-edges exist. The present work introduces a new method which overcomes the difficulties presented in the Vogler solution, when an arbitrary number of edges exists. The predictions are characterised by the same sensitivity as Vogler's solution while less computation time is needed for an accurate estimation.

Analysis: The Vogler solution is based on the Huygens principle [6] where the received signal strength, following single knife-edge diffraction, is calculated by adding all the contributions from the point sources which are assumed to exist in the open aperture above the knife-edge. This procedure is extended to the case of multiple edges using a formula expressed as multiple diffraction integrals. The evaluation of these is accomplished by the use of the repeated integrals of the error function [1]. In addition, the limits of each repeated integral are related to the size of the open region above the relevant knife-edge with the lower limit, denoted as β, related monotonically to the height of the diffracting edge and the upper limit is always +∞. When N knife-edges exist, the attenuation coefficient, normalised to free space loss, is calculated by [1]

$$A = \frac{1}{2^N} C_N e^{\sigma_N} \left(\frac{2}{\sqrt{\pi}}\right)^N \int_{\beta_1}^{\infty} \cdots \int_{\beta_N}^{\infty} e^{2f} e^{-x_1^2 - \dots - x_N^2} dx_1 \cdots dx_N \tag{1}$$

where the terms σ_N, C_N, f, β_M follow Vogler's notation [1]. As a final step, the multiple integral is transformed into a sum where a number of terms must be summed in order to find the correct result. Although the Vogler solution predicts the received power rapidly and accurately when the real parts of all the β-parameters, $\Re(\beta)$, are positive, the model becomes unreliable, and in most cases inaccurate, when $\Re(\beta)$ takes negative values, corresponding to edges which are below the line joining the preceding and succeeding edges. In these cases, the number of terms required in the series is also very large.

To overcome the difficulties and inaccuracies presented in the Vogler solution, a complete solution for arbitrary path profiles of N knife-edges has been developed. The key point is to transform the total problem into one which involves positive $\Re(\beta)$ everywhere. We denote the attenuation caused by n knife-edges by $V_{(N)}(S_1, \dots, S_N)$, where S_i denotes the aperture above the ith knife-edge. In this case, when M of the β-parameters have a negative real part and $(N–M)$ of the β-parameters have a non-negative real part, the final result is given by

$$A = \prod_{i=1}^{M} [1 - V_{(1)}(S_i')] \prod_{j=1}^{N-M} V_{(1)}(S_j) \tag{2}$$

where i indexes the edges with negative $\Re(\beta)$, j indexes the edges with non-negative $\Re(\beta)$, S' is the complementary surface (from $-\infty$ to the edge-top) and the following operation applies:

$$V_{(i)}(S_{n1}, S_{n2}, \dots, S_{ni}) \cdot V_{(j)}(S_{m1}, S_{m2}, \dots, S_{mj}) = V_{(i+j)}(S_{k1}, S_{k2}, \dots, S_{k(i+j)}) \tag{3}$$

All the apertures on the left-hand side of eqn. 3 are different to each other; also, at each operation the path geometry remains the same where, for a lower order diffraction process, some knife-edges cease to exist. Finally, the integration is carried through over the area of the knife-edge rather than the area above the edge, so the related repeated integral is not $i^N \mathrm{erfc}(\beta)$ but is equal to $(-1)^N i^N \mathrm{erfc}(-\beta)$.

To illustrate the above procedure, take a path profile with two knife-edges where both β-parameters have a negative real part. Following eqn. 2, the final result is obtained by the formula

$$A = -1 + \frac{1}{\sqrt{\pi}} e^{\beta_1'^2} \int_{\beta_1'}^{\infty} e^{-x^2} dx + \frac{1}{\sqrt{\pi}} e^{\beta_2'^2} \int_{\beta_2'}^{\infty} e^{-x^2} dx + \frac{1}{\pi} C_2 e^{\sigma_2} \int_{-\infty}^{\beta_1} \int_{-\infty}^{\beta_2} e^{2f} e^{-x_1^2 - x_2^2} dx_1 dx_2 \tag{4}$$

where the parameters C_2, f, σ_2, β_1, β_2 are as defined in [1], and

$$\beta_1' = \theta_1' \sqrt{\frac{ikr_1(r_2 + r_3)}{2(r_1 + r_2 + r_3)}} \text{ with } \theta_1' = \frac{h_1 - h_0}{r_1} + \frac{h_1 - h_3}{r_2 + r_3} \tag{5a}$$

$$\beta_2' = \theta_2' \sqrt{\frac{ik(r_1 + r_2)r_3}{2(r_1 + r_2 + r_3)}} \text{ with } \theta_2' = \frac{h_2 - h_0}{r_1 + r_2} + \frac{h_2 - h_3}{r_3} \tag{5b}$$

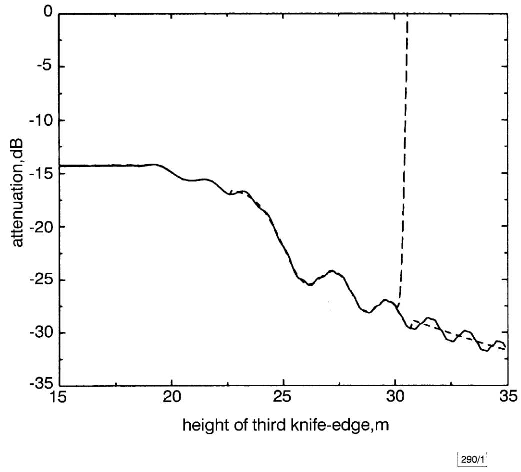

Fig. 1 *Attenuation function against knife-edge height*

—— original Vogler solution
– – – – preselect edges in Vogler solution
——— proposed solution

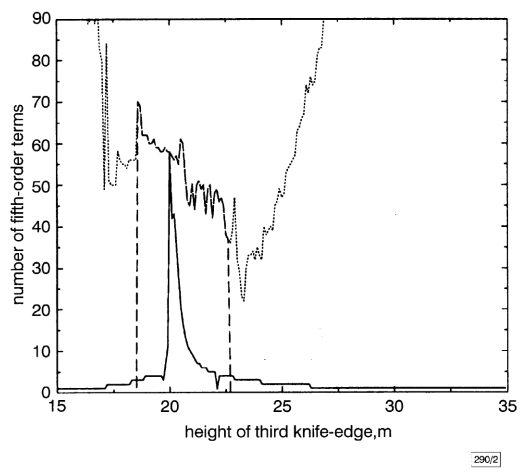

Fig. 2 *Number of terms against knife-edge height*

·········· original Vogler solution
– – – – preselect edges in Vogler solution
——— proposed solution

Example calculations: Fig. 1 shows an example comparison between Vogler and our solution, where five knife-edges exist. The frequency is 1GHz, $r_1 = r_2 = r_5 = r_6 = 15$m, $r_3 = r_4 = 10$m, $h_0 = ... = h_6 = 20$m, and finally, the height of the third knife-edge varies from 15 to 35m, in 0.1m steps, while obtaining the final results with 0.01dB accuracy.

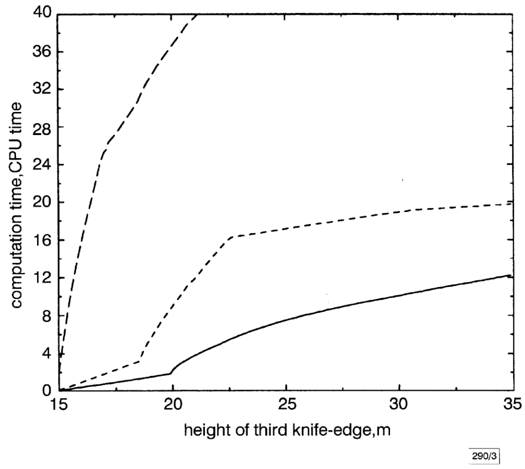

Fig. 3 *Comparison in computation time*

—— original Vogler solution
– – – – preselect edges in Vogler solution
——— proposed solution

Although the path geometry is relatively simple, the number of edges with negative $\Re(\beta)$ varies as the height of the third knife-edge increases. In Fig. 1, the results shown correspond to the original Vogler solution, to a Vogler solution where only the significant edges (with less negative $\Re(\beta)$) are considered and, finally, to the proposed method. Both the Vogler solutions fail to predict the correct result for $h_3 > 30$m where, in addition, the second Vogler solution presents the disadvantage of becoming insensitive to path profiles, hence, sliding to less accurate models [3].

In contrast, the new solution shows that, for $h_3 < 19$m, the solution reduces to a four knife-edge case. As the height of the middle knife-edge increases above 22m, the attenuation function result oscillates around the single knife-edge case as if only the middle edge existed in the path, as expected. In addition, the new solution needs a very small number of fifth-order terms for correct prediction at all heights and the series becomes extremely convergent since only one fifth-order diffraction term is required (Fig. 2).

Fig. 3 illustrates the total CPU computation time after each height-step, showing that with our solution the total CPU time for completing 200 simulations is minimum, compared to the other approaches, while predicting with most accuracy the attenuation function. This significant reduction in computation time is particularly important when area coverage predictions are being made using a large number of two-dimensional path profiles.

Conclusions: The model presented here allows rapid and highly accurate propagation predictions to be created from deterministic input data with low computational effort and without the need to avoid considering all the edges. In conjunction with both Vogler's solutions, it constitutes a reliable tool for any two-dimensional path profile when the knife-edge representation is appropriate. Finally, it may also be extended to three-dimensional truncated-edge solutions when Vogler's representation is applied which involves the numerical evaluation of the repeated integrals of the error function.

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