A PHYSICAL-STATISTICAL MODEL FOR LAND MOBILE SATELLITE PROPAGATION IN BUILT-UP AREAS

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INTRODUCTION

A physical-statistical model of land-mobile satellite propagation in built-up areas is derived from geometrical considerations and simple diffraction theory. The model has two components. The first predicts the probability with which a line-of-sight path exists between the mobile and the satellite. Models such as [1] predict the statistical fading characteristics of the received signal with the shadowing probability, *Ps*, as a parameter. This parameter is usually derived empirically by comparing measured results with the predictions from [1] and determining the value of *Ps* which leads to the best fit between predictions and measurements. In the new model *Ps* is related to physical parameters such as street widths and building height distributions. The second component of he model is a statistical prediction of the statistics of attenuation when a shadowed state is encountered, again related to physical parameters. The model allows predictions to be made for systems operated in areas where direct measurements are unavailable, or allows existing measurements to be scaled to apply to new parameter ranges. This is achieved with very low computational complexity and simple physical data.

GEOMETRY

The geometry of the situation to be analysed is illustrated in Figure 7. It describes a situation where a mobile is situated on a long straight street with the direct ray from the satellite impinging on the mobile from an arbitrary direction. The street is lined on both sides with buildings whose height varies randomly.
The parameters of this model are defined as follows:

ϕ - Elevation angle of the satellite from the mobile

θ - Azimuth angle of the satellite from the mobile relative to the axis of the street

w - Street width [m]

d_m - Perpendicular distance of the mobile from the building face [m]

h_m - The height of the phase centre of the mobile antenna above local ground level [m]

h_b - The height of the building immediately below the direct ray relative to local ground level [m]

h_r - The height of the direct ray above the building face relative to local ground level [m]

d_r - The distance along the direct ray from the mobile to the point on the ray immediately above the building.

Simple trigonometry yields the following relationships:

$$h_r = \begin{cases} h_m + \dfrac{d_m \tan\phi}{\sin\theta} & \text{for } 0 < \theta \le \pi \\ h_m + \dfrac{(w - d_m)\tan\phi}{\sin\theta} & \text{for } -\pi < \theta \le 0 \end{cases} \tag{1}$$

$$d_r = \begin{cases} \dfrac{d_m}{\sin\theta\cos\phi} & \text{for } 0 < \theta \le \pi \\ \dfrac{(w - d_m)}{\sin\theta\cos\phi} & \text{for } -\pi < \theta \le 0 \end{cases} \tag{2}$$

The signal may also include a contribution from a ray which reaches the mobile via a single reflection from the building face on the opposite side of the street. The equivalent parameters for the reflected ray are denoted by an extra subscript '*r*' and are given by:

$$h_{rr} = \begin{cases} h_m + \dfrac{(2w - d_m)\tan\phi}{\sin\theta} & \text{for } 0 < \theta \le \pi \\ h_m + \dfrac{(w + d_m)\tan\phi}{\sin\theta} & \text{for } -\pi < \theta \le 0 \end{cases} \tag{3}$$

$$d_{rr} = \begin{cases} \dfrac{(2w - d_m)}{\sin\theta\cos\phi} & \text{for } 0 < \theta \le \pi \\ \dfrac{(w + d_m)}{\sin\theta\cos\phi} & \text{for } -\pi < \theta \le 0 \end{cases} \tag{4}$$

SHADOWING PROBABILITY THEORY

In this model the direct ray is assumed to be the dominant source of power, so shadowing events are assumed to take place whenever this ray is shadowed. The ray is judged to be shadowed when the building height h_b exceeds some threshold height h_T relative to the direct ray height h_s at that point. The shadowing probability, P_s, can then be expressed in terms of the probability density function of the building height, $p_b(h_b)$ as:

$$P_s = \Pr(h_b > h_T) = \int_{h_T}^{\infty} p_b(h_b)\,dh_b \qquad (5)$$

It is then necessary to seek a suitable form for $p_b(h_b)$. Figure 1 shows a height distribution taken from geographical data for the City of Westminster [2], together with the results of fitting a Rayleigh distribution with parameter σ_b=15, 20 and 25m.

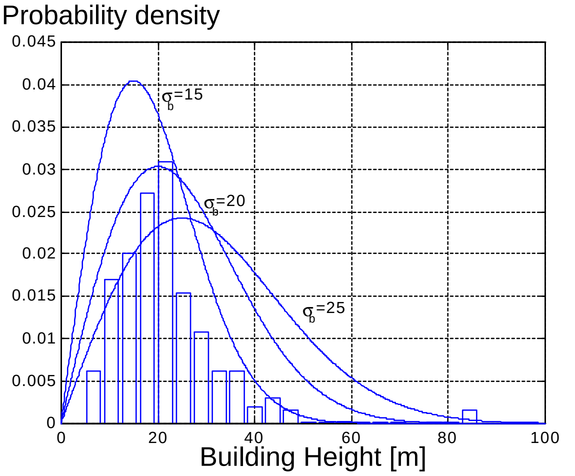

Figure 1: Building height distribution for the City of Westminster compared with a Rayleigh distribution

The fit is reasonable, although the Rayleigh distribution does tend to over-emphasise the effects of high buildings. The Rayleigh distribution has the particular advantage of analytic simplicity, since *Ps* can now be expressed as:

$$P_s = \int_{h_T}^{\infty} \frac{h_b}{\sigma_b^2} \exp\left(-\frac{h_b^2}{2\sigma_b^2}\right) dh_b = \exp\left(-\frac{h_T^2}{2\sigma_b^2}\right) \qquad (6)$$

The simplest definition of h_T is obtained by considering shadowing to occur exactly when the direct ray is geometrically blocked by the building face, ie h_T=h_r, with h_r given by (1) above.

In a more sophisticated approach the shadowing is considered to occur whenever a significant proportion (say 0.7) of the first Fresnel zone radius R_1 of the direct ray is obscured by the building. Given that the distance between the satellite and the building is very much greater than the mobile-building distance, R_1 is given by:

$$R_1 = \sqrt{\frac{\lambda d_m}{\sin\theta \cos\phi}} \qquad (7)$$

Then h_T is given by:

$$h_T = h_r - 0.7 R_1 \qquad (8)$$

where λ is the carrier wavelength.

These expressions are simple to calculate given the physical parameters and the building height distribution. In cases where an explicit distribution for the building height is unavailable, the parameter σ_b can be related to a qualitative classification of the environment. The same general approach can also be applied to shadowing by trees rather than buildings.

SHADOWING PROBABILITY RESULTS

In Figure 2 an example calculation of the shadowing probability from equation (6) with h_T given by h_T=h_r, with h_r calculated from (1) is shown, with parameters σ_b=15, w=35, d_m=w/2 and h_m=1.5. The model exhibits qualitatively reasonable behaviour, tending to 0 and 1 for elevation angles of 90° and 0° respectively, as expected. The azimuthal variation is more modest, but reduces P_s significantly when propagation occurs almost directly along the street.

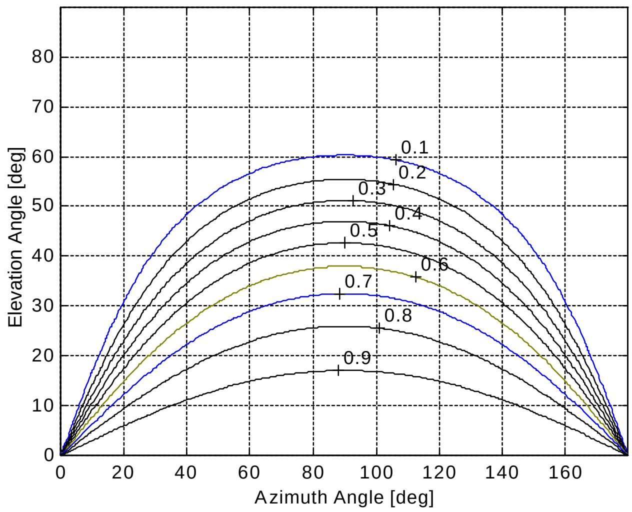

Figure 2: Shadowing Probability for σ_b=15, w=35, d_m=w/2, h_m=1.5

Figure 3 compares the model with measurements of P_s versus elevation angle in city and suburban environments taken from [3]. Here the model parameters are as in Figure 2, with θ fixed at 90°, as in the measurements. The model is seen to reproduce the important features of the measurements well.

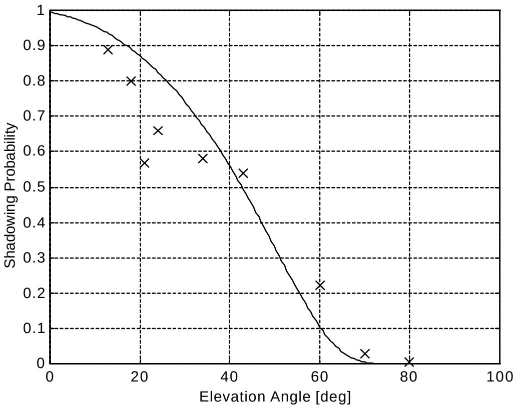

Figure 3: Comparison of theory and measurements

SHADOWING ATTENUATION THEORY

The approach adopted for attenuation is to apply single knife-edge diffraction theory for each of the rays propagating over the building. The individual rays are then power-summed, since it is desired to predict the local-median shadowing, rather than the fine structure of the multipath fading. The overall shadowed power received, *s*, can then be expressed as:

$$s = P(v_d) + \rho^2 P(v_r) \tag{9}$$

where ρ is the loss incurred on reflection, including effects due to polarization, surface roughness and the material reflection coefficient [2]. The diffraction parameters for the direct and reflected rays, v_d and v_r respectively are given by:

$$\begin{aligned} v_d &= (h_b - h_r)\sqrt{\frac{2}{\lambda d_r}} \\ v_r &= (h_b - h_r)\sqrt{\frac{2}{\lambda d_{rr}}} \end{aligned} \tag{10}$$

These expressions assume that the satellite-building distance is very much greater than the building-mobile distance and that the building height varies perpendicular to the ray. The diffracted power, $P(v)$ can be expressed in terms of the Fresnel cosine and sine integrals $C(v)$ and $S(v)$ as:

$$P(v) = \frac{1}{2}\left(\frac{1}{2} + C^2(v) - C(v) + S^2(v) - S(v)\right) \tag{11}$$

assuming only one building contributes to the diffraction process. For very low elevation angles the influence of multiple diffraction by buildings in adjacent streets may be accounted for using the model proposed in [4].

It is then desired to use the model to predict the statistics of the shadowing attenuation s_{dB} given the statistics of building heights from the previous section.

It can be shown from probability theory that the pdf of s_{dB} is:

$$p(s_{dB}) = p(h_b)\left|\frac{dh_b}{ds} \times \frac{s \ln 10}{10}\right| \tag{12}$$

where

$$\frac{ds}{dh_b} = \sqrt{\frac{2}{\lambda d_r}} P(v_d) \frac{dP}{dv}\bigg|_{v=v_d} + \rho^2 \sqrt{\frac{2}{\lambda d_{rr}}} P(v_d) \frac{dP}{dv}\bigg|_{v=v_r}$$

and $\frac{dP}{dv}$ can be calculated numerically with ease. There are technical difficulties associated with applying (12) to functions which are multivalued, so it is assumed subsequently that $P(v)$ remains at 0dB once v goes below -0.77. The probability density function in (12) can then be integrated to provide cumulative statistics.

SHADOWING ATTENUATION RESULTS

The result of (9) is illustrated in Figure 4 as a function of the elevation angle for a reflection loss of 10dB, h_b=15m and other parameters as before. The effect of the reflected ray is clearly visible.

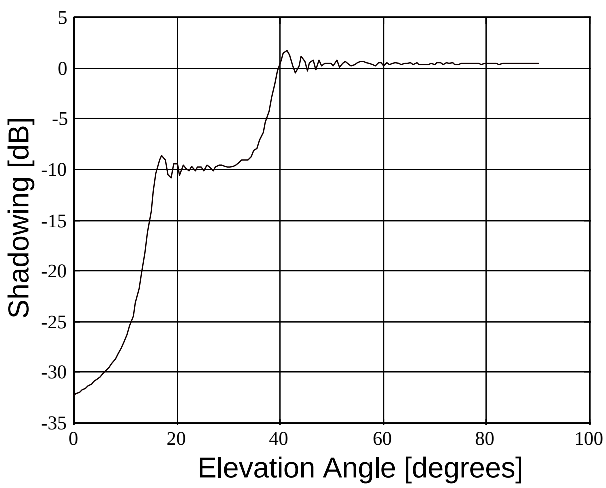

Figure 4: Shadowing Attenuation versus elevation angle

For an elevation angle of 40°, predictions from (11) for the variation of attenuation with building height for various frequencies are shown in Figure 5. This will translate to a change in the dynamics of attenuation as the mobile moves down the street and has important implications for processes such as power control.

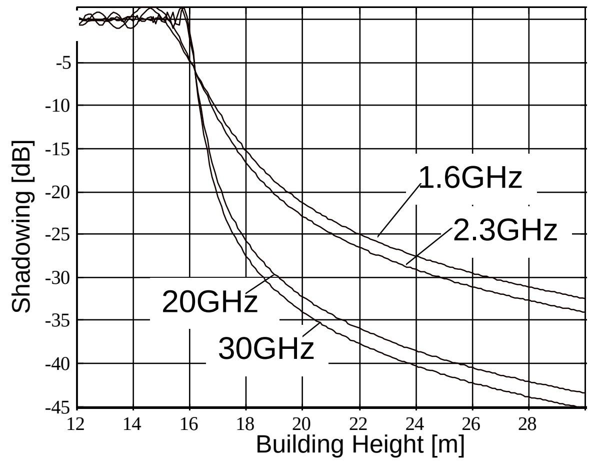

Figure 5: Shadowing dynamics versus frequency

An example computation from (12) is shown in Figure 6 for various elevation angles. In this case, the reflected path is assumed negligible (which may occur if, for example, the street has buildings on one side only). The results are qualitatively reasonable and have been verified using a Monte Carlo simulation.

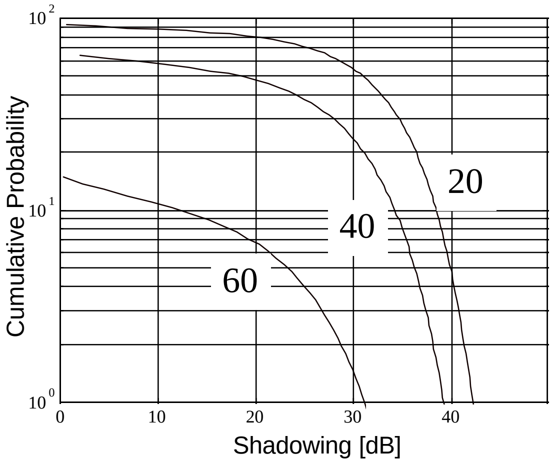

Figure 6: Shadowing probability, ϕ=20°,40°,60°

Relatively little measured data exists for the case where shadowing occurs mainly as a result of buildings rather than trees. However, ongoing work will compare this model to data obtained from measurement campaigns conducted at the University of Surrey.

CONCLUSIONS

The model presented here allow mobile-satellite propagation predictions to be assigned parameters and values which are directly related to the physical environment and which vary with both azimuth and elevation angles as would be expected in a real system. This is important particularly for predicting the performance of multiple satellite diversity systems. The physical-statistical nature of the model allows useful predictions to be obtained from objective input data with very modest computational effort.

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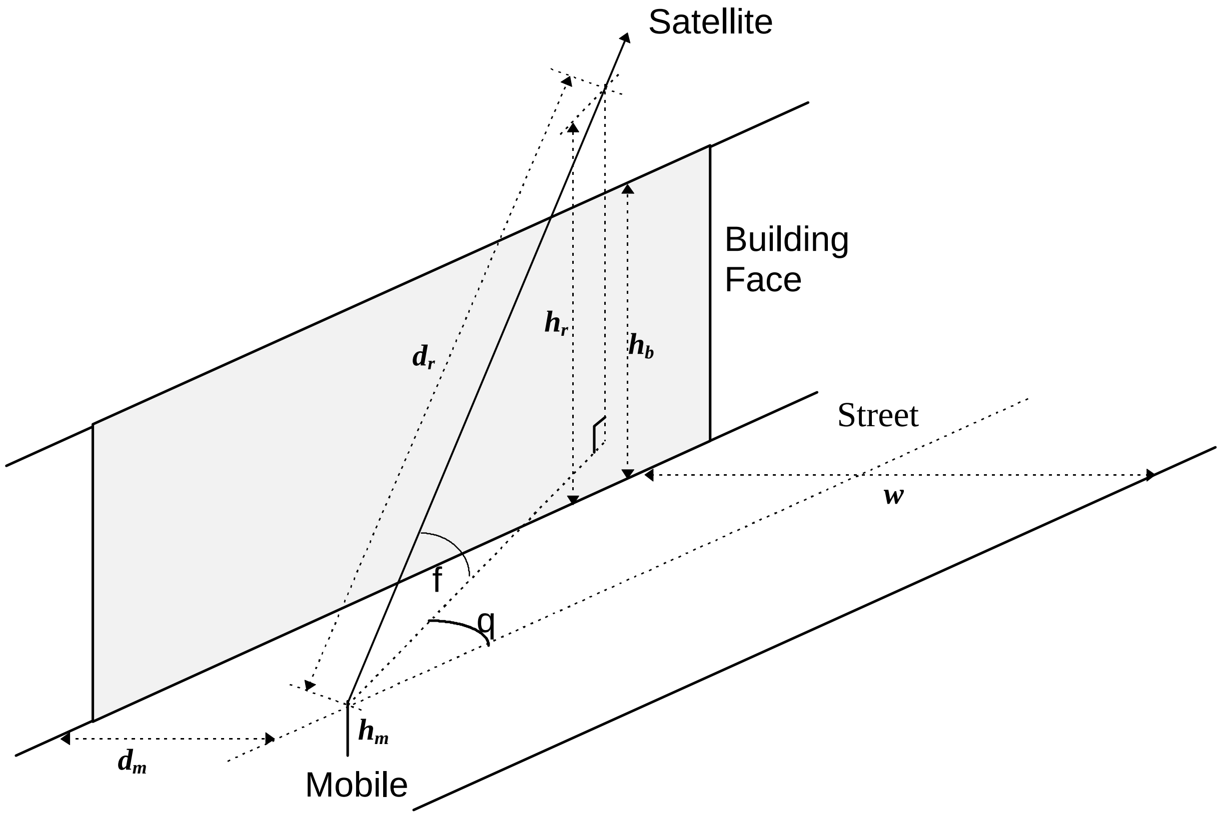

Figure 7: Mobile / Satellite / Street Geometry