Physical model of shadowing probability for land mobile satellite propagation

S.R. Saunders and B.G. Evans

Indexing terms: Mobile radio systems, Mobile satellite communication, Radiowave propagation

A physical model of shadowing probability for a land-mobile satellite system is derived from a simple geometrical model. The resulting model exhibits physically reasonable behaviour, in good agreement with empirical measurements and allows the variation of shadowing states with both satellite elevation and azimuth to be predicted.

Introduction: For the prediction of the propagation characteristics of land-mobile satellite (LMS) Systems, an important parameter is the probability with which a line-of-sight path exists between the mobile and the satellite. Models such as that in [1] predict the statistical fading characteristics of the received signal with the shadowing probability P_S as a parameter. This parameter is usually derived empirically by comparing measured results with the predictions from [1] and determining the value of P_S which leads to the best fit between predictions and measurements. In this Letter the situation is analysed geometrically to determine P_S in terms of physical parameters such as street widths and building height distributions. This allows predictions to be made for systems which operate in areas where direct measurements are unavailable, or allows existing measurements to be scaled to apply to new parameter ranges.

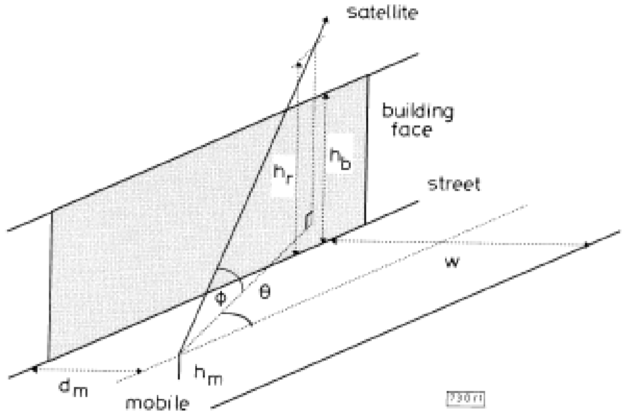

Fig. 1 *Mobile/satellite/street geometry*

Analysis: The geometry of the situation to be analysed is illustrated in Fig. 1. It describes a situation where a mobile is situated on a long straight street with the direct ray from the satellite impinging on the mobile from an arbitrary direction. The street is lined on both sides with buildings whose height varies randomly. The parameters of this model are defined as follows: ϕ is the elevation angle of the satellite from the mobile, θ the azimuth angle of the satellite from the mobile relative to the axis of the street, w the street width [m], d_M the perpendicular distance of the mobile from the building face [m], h_M the height of the phase centre of the mobile antenna above local ground level [m], h_B the height of the building immediately below the direct ray relative to local ground level [m], and h_R the height of the direct ray above the building face relative to local ground level [m].

The direct ray is judged to be shadowed when the building height h_B exceeds some threshold height h_T relative to the direct ray height h_S at that point. The shadowing probability P_S can then be expressed in terms of the probability density function of the building height $p_B(h_B)$ as

$$P_s = \Pr(h_b > h_T) = \int_{h_T}^{\infty} p_b(h_b)dh_b \qquad (1)$$

It is then necessary to seek a suitable form for $p_B(h_B)$. Fig. 2 shows a height distribution taken from geographical data for the City of Westminster [2], together with the results of fitting a Rayleigh distribution with parameter σ_B = 15, 20 and 25 m.

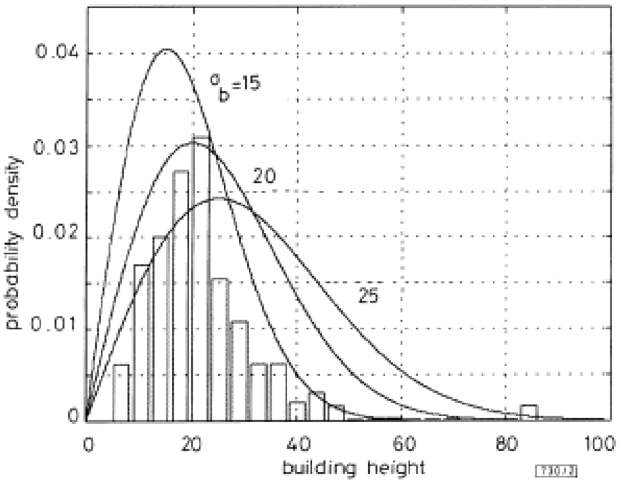

Fig. 2 *Building height distribution for the City of Westminster compared with Rayleigh distribution*

The fit is reasonable, although the Rayleigh distribution tends to over-emphasise the effects of high buildings. The Rayleigh distribution has the particular advantage of analytic simplicity, since P_S can now be expressed as

$$P_s = \int_{h_T}^{\infty} \frac{h_b}{\sigma_b^2}\exp\left(-\frac{h_b^2}{2\sigma_b^2}\right)dh_b = \exp\left(-\frac{h_T^2}{2\sigma_b^2}\right) \qquad (2)$$

The simplest definition of h_T is obtained by considering shadowing to occur exactly when the direct ray is geometrically blocked by the building face. Simple trigonometry applied to this yields the following expression for h_T:

$$h_T = h_r = \begin{cases} h_m + \frac{d_m \tan\phi}{\sin\theta} & \text{for } 0 < \theta \leq \pi \\ h_m + \frac{(w-d_m)\tan\phi}{\sin\theta} & \text{for } -\pi < \theta \leq 0 \end{cases} \qquad (3)$$

In a more sophisticated approach the shadowing is considered to occur whenever a significant proportion (say 0.7) of the first Fresnel zone radius R_1 of the direct ray is obscured by the building. Given that the distance between the satellite and the building is very much greater than the mobile-building distance, R_1 is given by

$$R_1 = \sqrt{\frac{\lambda d_m}{\sin\theta\cos\phi}} \qquad (4)$$

Then h_T is given by

$$\begin{aligned} h_T &= h_r - 0.7R_1 \\ &= \begin{cases} h_m + \frac{d_m\tan\phi}{\sin\theta} - 0.7\sqrt{\frac{\lambda d_m}{\sin\theta\cos\phi}} & \text{for } 0 < \theta \leq \pi \\ h_m + \frac{(w-d_m)\tan\phi}{\sin\theta} - 0.7\sqrt{\frac{\lambda d_m}{\sin\theta\cos\phi}} & \text{for } -\pi < \theta \leq 0 \end{cases} \end{aligned} \qquad (5)$$

where λ is the carrier wavelength.

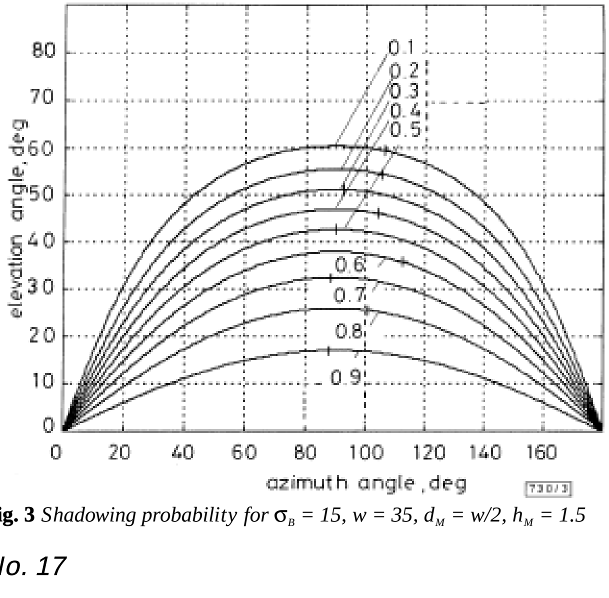

Fig. 3 *Shadowing probability for* σ_B *= 15, w = 35,* d_M *= w/2,* h_M *= 1.5*

These expressions are simple to calculate given the physical parameters and the building height distribution. In cases where an explicit distribution for the building height is unavailable, the parameter σ_B can be related to a qualitative classification of the environment. The same general approach can also be applied to shadowing by trees rather than buildings.

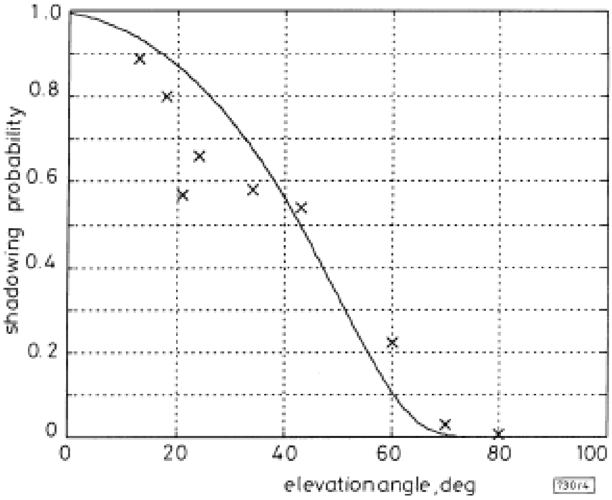

Fig. 4 *Comparison of theoretical (parameters as in Fig. 3, $\theta = 90°$) and empirical results taken from [3] for shadowing probability against elevation angle in city/suburban environment*

Results: In Fig. 3 an example calculation of the shadowing probability from eqn. 2 with h_T given by eqn. 3 is shown, with parameters $\sigma_B = 15$, $w = 35$, $d_M = w/2$ and $h_M = 1.5$. The model exhibits qualitatively reasonable behaviour, tending to 0 and 1 for elevation angles of 90 and 0°, respectively, as expected. The azimuthal variation is more modest, but reduces P_S significantly when propagation occurs almost along the street.

Fig. 4 compares the model with measurements of P_S against elevation angle in city and suburban environments taken from [3]. Here the model parameters are as in Fig. 3, with θ fixed at 90°, as in the measurements. The model is seen to reproduce the important features of the measurements well.

Conclusions: The model presented here allows two state models of LMS propagation such as [1] to be assigned a shadowing probability which is directly related to the physical environment and which varies with both azimuth and elevation angles as would be expected in a real system. Such a model is particularly of use when predicting the performance for systems which rely on satellite dual-diversity where azimuth variations play a crucial role. It is also useful for inferring the variation of shadowing with azimuth from measurements taken at a fixed azimuth angle. Most significantly, it allows LMS system performance in suburban areas to be related to an objective measure of the environment, namely the building height distribution.

20 June 1996

Electronics Letters Online No: 19961058

S.R. Saunders and B.G. Evans (*Centre for Satellite Engineering Research, University of Surrey, Guildford, Surrey GU1 2SA, United Kingdom*)

References

1 LUTZ, E., CYGAN, D., DIPPOLD, M., DOLAINSKY, F., and PAPKE, W.: ‘The land mobile satellite communication channel - Recording, statistics and channel model’, *IEEE Trans.*, 1991, **VT-40**, (2), pp. 375–386

2 SAUNDERS, S.R.: ‘Diffraction modelling of mobile radio wave propagation in built-up areas’. Brunel University PhD thesis, 1991, p.172

3 PARKS, M.A.N., EVANS, B.G., and BUTT, G.: ‘High elevation angle propagation results, applied to a statistical model and an enhanced empirical model’, *Electron. Lett.*, 1993, **29**, (19), pp. 1723–1725