A PHYSICAL-STATISTICAL PROPAGATION MODEL FOR DIVERSITY IN MOBILE SATELLITE PCN

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Abstract - This paper proposes a physical-statistical model of satellite-diversity, for particular application to non-geostationary satellite PCN systems operated in built-up areas. The output of the model is an explicit formulation for the correlation between shadowing states associated with pairs of satellites, which can be used as input for Markov models of the satellite channel. Results are presented for a number of practical cases and are found to be in excellent agreement with analysis of real buildings.

I. INTRODUCTION

All of the proposed mobile satellite PCN systems use satellite diversity as a key means of increasing availability. Accurate prediction of the effect of this diversity is essential in order to correctly optimise such systems and to predict their availability. This paper proposes a physical-statistical model of satellite-diversity, for particular application to built-up areas. The new model takes as its starting point a previous model of shadowing probability [1] and shadowing attenuation [2] by the authors, extended to predict the correlation between mobile-satellite channels, separated in both azimuth and elevation.

Physical-statistical methods have been invented by the authors as a novel approach to propagation modelling which has practical benefits for the system designer, and has been shown [2] to yield accurate and consistent predictions. The starting point for a physical-statistical model consists of electromagnetic and physical analysis of a canonical situation which includes all of the major propagation mechanisms. This canonical analysis is then performed on a statistical input data set, yielding a distribution of the output predictions. The predictions are then not linked to specific locations, but rather represent the distribution of the output parameter over the ensemble of possible locations. Such predictions are particularly appropriate forglobal satellite systems, where the system designer wishes to predict performance over whole continents, which would be quite impossible with purely deterministic approaches. Physical-statistical models therefore require only simple input data such as input distribution parameters (e.g. mean building height, building height variance). The environment description is entirely objective, avoiding problems of subjective classification. The models are based on sound physical principles, so they are applicable over very wide parameter ranges. Finally, by precalculating the effect of specific input distributions, the required computational effort can be very small.

The new model begins by proposing a canonical geometry for a mobile operated in a street canyon, composed of buildings on both sides of the street in which the mobile is located. This geometry is parameterised by physical parameters such as building height and street width, avoiding the need for imprecise and ambiguous classifications such as urban and suburban. Analysis of the resulting geometry for two distinct mobile-satellite channels yields explicit expressions for the correlation between the shadowing states of any pair of satellites.

The expression for the shadowing correlation is intended for use in models such as [3] and [4], which characterise the channels in terms of a number of distinct states, with appropriate transition probabilities between the states. The new expression enables the transition probabilities to be related directly to the known physical parameters of the system, such as the elevation and azimuth angles of the satellites and the building heights and street widths around the mobile.

II. CANONICAL GEOMETRY

The Markov model proposed in [4] and demonstrated in [7] uses 4 states to represent the discrete shadowing states of a system involving two satellites. In order to use the model, the transition probabilities between all states are needed, and these in turn depend on the correlation coefficient between the states. It was suggested in [4] that the correlation

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coefficient could be obtained either from measurements or from analysis of fish-eye lens photography [5]. The disadvantage with these approaches is that the result depends strongly on the particular environments chosen for analysis, and does not make explicit the dependence of the correlation on the physical parameters.

In our approach, this is remedied via analysis of the canonical street canyon geometry shown in Figure 4. This is the same geometry which has been shown to give good agreement with measurements when predicting the probability of shadowing for a single satellite [1] and for predicting the shadowing attenuation [2]. The geometry describes a situation where a mobile is situated on a long straight street with the direct ray from the satellite impinging on the mobile from an arbitrary direction. The street is lined on both sides with buildings whose height in this work is considered constant, although in previous work ([1] and [2]) it was allowed to vary randomly.

The parameters of this model are defined as follows:

ϕ - Elevation angle of the satellite from the mobile

θ - Azimuth angle of the satellite from the mobile relative to the axis of the street

w - Street width [m]

d_m - Perpendicular distance of the mobile from the building face [m]

h_m - The height of the phase centre of the mobile antenna above local ground level [m]

h_b - The height of the building immediately below the direct ray relative to local ground level [m]

h_r - The height of the direct ray above the building face relative to local ground level [m]

Simple trigonometry yields the following relationships:

$$h_r = \begin{cases} h_m + \dfrac{d_m \tan\phi}{\sin\theta} & \text{for } 0 < \theta \le \pi \\ h_m - \dfrac{(w - d_m)\tan\phi}{\sin\theta} & \text{for } -\pi < \theta \le 0 \end{cases} \qquad (1)$$

In previous work the model was also extended to include a contribution from a ray which reaches the mobile via a single reflection from the building face on the opposite side of the street. In this case, however, the shadowing state is considered to be defined only by the direct ray, which is expected to represent the dominant source of power.

The direct ray is judged to be shadowed when the building height h_b exceeds some threshold height h_T relative to the direct ray height h_s at that point. The simplest definition of h_T is obtained by considering shadowing to occur exactly when the direct ray is geometrically blocked by the building face, i.e. $h_{T=} h_R$.

In a more sophisticated approach the shadowing is considered to occur whenever a significant proportion (say 0.7) of the first Fresnel zone radius R_1 of the direct ray is obscured by the building, but this is considered unnecessary for the purposes of the current work.

When two satellites are present, their locations are defined by two pairs of elevation and azimuth angles (θ_1,ϕ_1) and (θ_2,ϕ_2). The shadowing states of the satellite i is defined as:

$$S_i = \begin{cases} 0 \text{ if } h_b \ge h_r \text{ (bad channel state)} \\ 1 \text{ if } h_b < h_r \text{ (good channel state)} \end{cases} \qquad (2)$$

where (θ_i,ϕ_i) is used in (1) to find the corresponding value for h_r.

The correlation between these shadowing states is then defined by:

$$\rho = \frac{E\left[(S_1 - \overline{S}_1)\cdot(S_2 - \overline{S}_2)\right]}{\sigma_1 \cdot \sigma_2} = \frac{1}{\sigma_1 \cdot \sigma_2}\cdot\left[E(S_1 S_2) - \overline{S}_1 \cdot \overline{S}_2\right]$$

where $E[\ .\ ]$ denotes expectation and $E[S] = \overline{S}$. In this work, the task is to find this correlation, where the expectation is taken over all possible azimuth angles relating to the street. This is justified because the predictions sought should be applicable to the ensemble of all possible streets with corresponding building parameters. Since streets have no particular orientation with respect to the orbital path of a satellite constellation, it will be assumed that the azimuth angle of the street is uniformly distributed on $[0,2\pi]$.

III. ANALYSIS

Under the assumptions given in Section II, the mean value for the shadowing state is given by

$$\overline{S}_i = \int_0^{2\pi} S_i(\varphi_s) p(\varphi_s) d\varphi_s = \frac{1}{2\pi}\int_0^{2\pi} u_i(h_r - h_b) d\varphi_s \qquad (3)$$

where u_i is the step function

$$u_i(h_r - h_b) = \begin{cases} 1 & h_r - h_b \ge 0 \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

Hence, the mean signal amplitude is described by the equation

$$\overline{S}_i = \frac{1}{2\pi}\left\{\int_0^{\pi} u_i(h_r - h_b) d\varphi_s + \int_{\pi}^{2\pi} u_i(h_r - h_b) d\varphi_s\right\} \qquad (5)$$

For simplicity, the following notation will be used:

$$a_i \equiv \arcsin\left(\frac{d_m \tan\vartheta_i}{h_b - h_m}\right), \quad b_i \equiv \arcsin\left(\frac{(w - d_m)\tan\vartheta_i}{h_b - h_m}\right)$$

The terms inside the integral in (5) are equal to unity when,

for $0 < \varphi_s \le \pi$,

$$h_r \geq h_b \Rightarrow \sin(\varphi_s) \leq \frac{d_m \tan\vartheta_i}{h_b - h_m} \Rightarrow 0 \leq \varphi_s \leq a_i \text{ and } \pi - a_i \leq \varphi_s \leq \pi \quad (6a)$$

and also when, for $-\pi < \varphi_s \leq 0$,

$$h_r \geq h_b \Rightarrow \sin(\varphi_s) \geq -\frac{(w - d_m)\tan\vartheta_i}{h_b - h_m} \Rightarrow \varphi_s \geq -b_i \text{ and } -\pi \leq \varphi_s \leq -\pi + b_i \quad (6b)$$

Therefore, the mean state is given by

$$\overline{S_i} = \frac{1}{2\pi}\left\{\int_0^{a_i} d\varphi_s + \int_{\pi-a_i}^{\pi} d\varphi_s + \int_{\pi}^{\pi+b_i} d\varphi_s + \int_{2\pi-b_i}^{2\pi} d\varphi_s\right\} = \frac{1}{\pi}(a_i + b_i) \quad (7)$$

The term $E[S_1 S_2]$ is given by:

$$E[S_1 S_2] = \int_0^{2\pi} S_1 S_2\, p(\varphi_s)\, d\varphi_s = \frac{1}{2\pi}\int_0^{2\pi} S_1 S_2\, d\varphi_s \quad (8)$$

By substituting the signal amplitudes in (8), it becomes:

$$E(S_1 S_2) = \frac{1}{2\pi}\left\{\begin{array}{l}\int_0^{\pi} u_1(h_r - h_b)\cdot u_2(h_r - h_b)\, d\varphi_s \\ + \int_{\pi}^{2\pi} u_1(h_r - h_b)\cdot u_2(h_r - h_b)\, d\varphi_s\end{array}\right\} \quad (9)$$

or,

$$E(S_1 S_2) = \frac{1}{2\pi}[I_1 + I_2] \quad (10)$$

where

$$I_1 = \int_0^{\pi} u_1[h_r - h_b]\cdot u_2[h_r - h_b]\, d\varphi_s \quad (11)$$

$$I_2 = \int_{\pi}^{2\pi} u_1[h_r - h_b]\cdot u_2[h_r - h_b]\, d\varphi_s \quad (12)$$

Each integral is split into two integrals, since u_2 changes value within the integration limits, so by setting $\varphi_1 = 0$, without any loss of generality, we have

$$I_1 = I_{11} + I_{12} \quad (13)$$

where

$$I_{11} = \int_0^{\pi-\Delta\varphi} u_1\left[h_m + \frac{d_m \tan\vartheta_1}{\sin(\varphi_s)} - h_b\right]\cdot u_2\left[h_m + \frac{d_m \tan\vartheta_2}{\sin(\Delta\varphi + \varphi_s)} - h_b\right] d\varphi_s$$

$$I_{12} = \int_{\pi-\Delta\varphi}^{\pi} u_1\left[h_m + \frac{d_m \tan\vartheta_1}{\sin(\varphi_s)} - h_b\right]\cdot u_2\left[h_m - \frac{(w - d_m)\tan\vartheta_2}{\sin(\Delta\varphi + \varphi_s)} - h_b\right] d\varphi_s$$

and $\Delta\varphi = |\varphi_1 - \varphi_2|$.

As regards the term I_{11}, the final result depends on the values of the physical parameters, so many different cases exist. In order to show how these terms can be solved we illustrate the expression of I_{11} when $\Delta\varphi > a_1$. In this case, the integral expression reduces to

$$\int_0^{z_1} u_2\left[h_m + \frac{d_m \tan\vartheta_2}{\sin(\varphi_s + \Delta\varphi)} - h_b\right] d\varphi_s = e_1 + e_2 \quad (16)$$

where

$$e_1 = \begin{cases} a_2 - \Delta\varphi & \Delta\varphi < a_2 < z_1 + \Delta\varphi \\ z_1 & a_2 \geq z_1 + \Delta\varphi \\ 0 & a_2 < \Delta\varphi \end{cases} \quad (16a)$$

and

$$e_2 = \begin{cases} z_1 + \Delta\varphi - \pi + a_2 & \pi - a_2 \geq \Delta\varphi \\ z_1 & \pi - a_2 < \Delta\varphi \end{cases} \quad (16b)$$

and the term e_2 is evaluated only when $z_1 + \Delta\varphi > \pi - a_2$. Additionally, the term z_1 represents the minimum of a_1 and $\pi - \Delta\varphi$. Similar expressions exist for the other cases resulting in a complicated solution.

Following the same approach for I_2,

$$I_2 = I_{21} + I_{22} \quad (17)$$

where

$$I_{21} = \int_{\pi}^{2\pi-\Delta\varphi} u_1\left[h_m - \frac{(w - d_m)\tan\vartheta_1}{\sin(\varphi_s)} - h_b\right]\cdot u_2\left[h_m - \frac{(w - d_m)\tan\vartheta_2}{\sin(\Delta\varphi + \varphi_s)} - h_b\right] d\varphi_s$$

$$I_{22} = \int_{2\pi-\Delta\varphi}^{2\pi} u_1\left[h_m - \frac{(w - d_m)\tan\vartheta_1}{\sin(\varphi_s)} - h_b\right]\cdot u_2\left[h_m + \frac{d_m \tan\vartheta_2}{\sin(\Delta\varphi + \varphi_s)} - h_b\right] d\varphi_s$$

Again, the final result for I_2 depends on the values for the physical parameters.

A common conclusion is that the final value of $E(S_1 S_2)$ and hence, ρ, depends only on the difference of the azimuth angles of the two satellites and not their absolute values. This has now been formally proven by parameterising the above expression using only $\Delta\varphi$. The standard deviations in (2) are determined by:

$$\sigma_i = \sqrt{E(S_i^2) - E^2(S_i)} = \sqrt{E(S_i) - E^2(S_i)} \quad (18)$$

Consequently, the correlation coefficient is calculated by (2), (7) and expressions such as (16,16a,16b).

IV. RESULTS

The behaviour of the new model will be illustrated here via several examples for particular physical parameters.

Example 1

First the mean value of the shadowing state is examined for a single satellite, taking the parameters as follows: $h_m = 5\text{m}$, $h_b = 20\text{m}$, $\vartheta = 30°$. Figure 1 shows the mean state as a function of the street width for three values of d_m. Note the following points:

- For small street widths, the good state probability increases with width because the probability of being blocked by the buildings on at least one side of the street is decreasing.
- When the street width exceeds a threshold value the shadowing probability becomes essentially constant. This is because the path to the satellite is then always clear of the buildings on one side of the street, while the other path remains blocked for some azimuth angles for fixed d_m. In the case $d_m = w/2$ both sides of the street become clear for all azimuth angles simultaneously.
- For fixed street widths, less than the threshold value, it is better to be well away from the centre of the street, i.e. d_m should be small.
- For widths greater than the threshold value, the best situation is $d_m = w/2$.

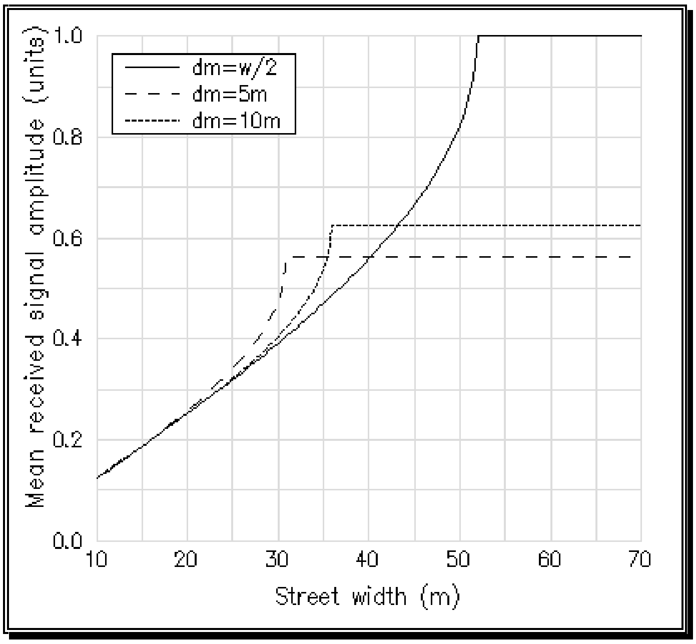

Figure 1: Mean value vs Street width

Example 2

In Figure 2 the mean shadowing state is plotted versus the satellite elevation angle, with $h_m = 5m$, $h_b = 20m$ and $d_m = 5m$ for two street widths. Points to note here are:

- Both curves go to 1 since the satellite is eventually unobstructed for all azimuth angles.
- The non-symmetric case is larger since the (w-dm) contributes more and so it is better to have such a situation (no symmetry on how the mean value is obtained)
- In general the curves exhibit three regions. The first occurs for low elevation angles, where buildings both sides of the street contribute. At intermediate elevation angles the more distant building never shadows. Finally, the unobstructed case is encountered. In the case $d_m = w/2$ the intermediate region is of zero extent.

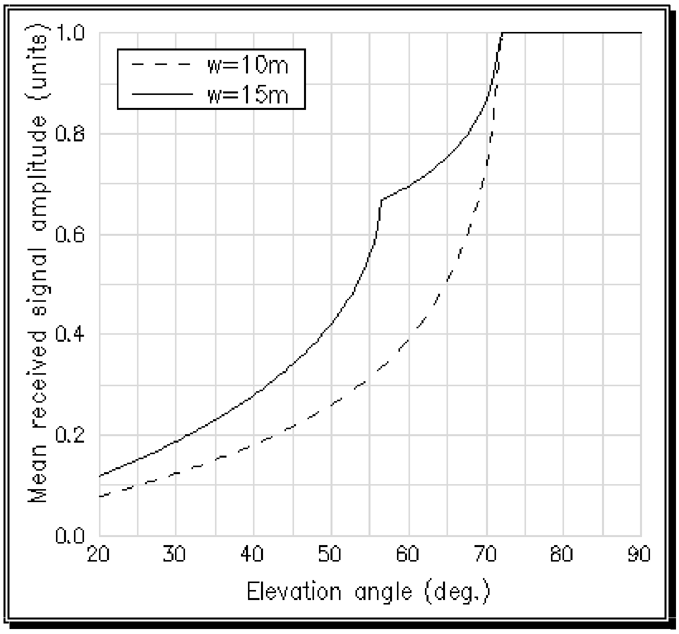

Figure 2: Mean value vs Elevation angle

Example 3

In Figure 3 calculations from the correlation coefficient model are presented, versus the azimuth separation between the satellites for three pairs of elevation angles and with $h_b = 20m$, $h_m = 5m$ and $d_m = 5m$. Points to note are:

- For azimuth separations less than around 30°, significant positive correlation is observed because both satellite paths traverse the same building for most azimuth angles.
- If the elevation angles are different, the correlation coefficient is reduced. However, the level of this correlation does not only depend on the difference between the elevation angles. For larger differences, the absolute value of correlation tends to be smaller.
- For azimuth separation between around 30° and 140°, significant negative correlation is observed, peaking in the region of 90°. This corresponds to the situation where one path is often over a building while the other is roughly aligned with the direction of the street canyon.
- For separations between around 140° and 180°, weaker positive correlation is observed, corresponding to the case where the paths cross buildings on opposite sides of the street or are both essentially unshadowed due to alignment with the street. This would also depend on the correlation between building heights on opposite sides of the street.

Figure 3 should be compared with Figure 6 in [7], where results have been extracted from fish-eye photographs. The results are qualitatively extremely similar, with residual variations due only to the specific nature of the buildings in the photograph analysed. A different situation is presented in Figure 5 of [7], but the case of Figure 6 is encountered much more frequently in practice. This verifies that the street canyon model is a useful and representative model of reality in urban areas.

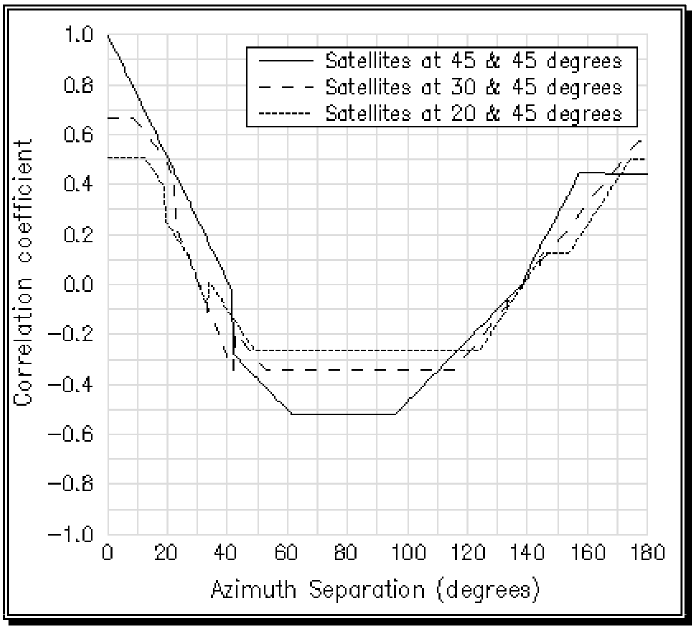

Figure 3 Correlation coefficient for different azimuth separations

V. CONCLUSIONS & FURTHER WORK

Key conclusions arising from this work are:

- The assertion in [4] that shadowing correlation depends only the azimuth separation between satellites, rather than the absolute values, has been formally proved subject to the assuptions used here.
- For typical urban situations, significant satellite diversity can be obtained for azimuth separations between 30° and 140°.

Further work will include:
- Extension to allow building height and width to vary randomly

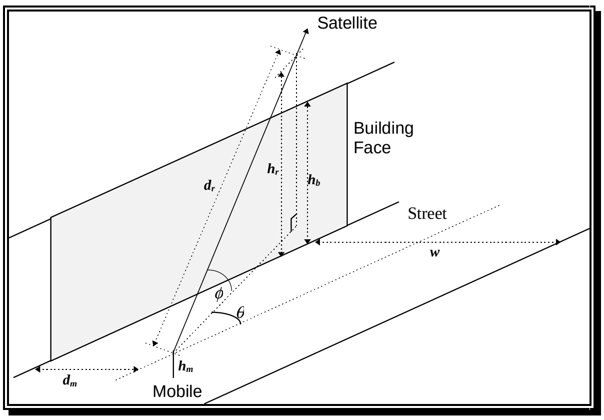

Figure 4 ***Canonical Street Canyon Geometry***

- Extension to allow *continuous* rather than *discrete* prediction of correlated shadowing attenuation, using the basic one-satellite model described in [2].

Overall this model combined with the approach [7] gives a rapid and accurate computation of non-geostationary mobile satellite system performance.

VI. REFERENCES

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